Suppression of Undesired Inputs of Linear Systems by Eigenspace Assignment

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In this paper, a method using output feedback is proposed to suppress the response of linear systems to undesired inputs and, in particular, to reduce the vibration response of flexible structures to these inputs. This method does not need to measure undesired inputs (or external forces). The analysis assumes that the location of the undesired inputs are known, although the general time dependency is unknown. The feedback gain matrix is calculated to assign the eigenvalues and left-hand eigenvectors of the closed-loop system to specified values. The effect of the undesired inputs on a closed-loop system can be altered or significantly reduced by properly choosing the left-hand eigenvectors of the system. The stability of the control system is guaranteed by properly choosing the output matrix, which can decouple the controlled modes from the uncontrolled modes. An example of forced vibration of a simple flexible structure is presented to demonstrate the proposed method.

Nomenclature

$[A]$ = $(N \times N)$ system matrix
$[B]$ = $(N \times m)$ input matrix
$[D]$ = $(r \times N)$ output matrix
$[E]$ = $N \times n)$ external force matrix
$[F]$ = $(m \times r)$ feedback gain matrix
$\{f(t)\}$ = external force
$[H][\hat{P}][T][Z]$ = matrices, see Eqs. (18–23)
$\{h_i\}$ = ith column of $[H]$
m = number of controls
N = order of the state model of linear system
n = number of external forces
$\{p_i\}$ = combination vector between left eigenvectors of open-loop and closed-loop system
r = number of outputs
$\{u(t)\}$ = control vector
$\{x(t)\}$ = state vector
$\{y(t)\}$ = output vector
$[\Lambda]$ = diag $\{\lambda_i\}$ eigenvalue matrix of $[A]$
$[\Lambda^{(d)}]$ = diag $\{\lambda_i^{(d)}\}$ desired eigenvalue matrix
$[\Phi]$ = right eigenvector matrix of $[A]$
$[\Phi_C]$ = submatrix $[\Phi]$ corresponding to controlled modes

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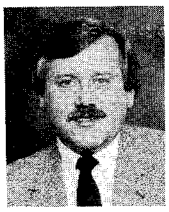

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$[\Phi_U]$ = submatrix of $[\Phi]$ corresponding to uncontrolled modes
$[\Phi_1]$ = $(N \times q)$ submatrix of $[\Phi]$
$\{\phi_i\}$ = ith column of $[\Phi]$
$[\Psi]$ = left eigenvector matrix of $[A]$
$[\Psi^{(d)}]$ = desired left eigenvector matrix
$[\Psi_1]$ = $(N \times q)$ submatrix of $[\Psi]$, see Eq. (10)
$\{\psi_i\}$ = ith column of $[\Psi]$
$\{\psi_i^{(d)}\}$ = ith column of $[\Psi^{(d)}]$
[ ] = matrix
{ } = vector
$[\ ]^T$ = transpose of matrix
$[\ ]^{-1}$ = inverse of matrix
$[\ ]^{\dagger}$ = pseudoinverse of matrix

Introduction

THE objective of most active control techniques applied to flexible structures is to alter the transient response of the structure for given initial conditions. In contrast with the free vibration problem, the control of forced vibration has not been fully investigated. The intent of the control for forced vibration is to suppress or reduce the effect of external forces on the structure. The external forces, however, are, in general, time varying and unpredictable. If the external force is known or has known statistical properties such as zero mean white noise, there exist various analytical approaches to handle the forced response control problem.[1] However, for structural systems, external forces rarely have this particular nature and, in addition, it is very difficult to measure them with good accuracy.

In this paper we consider, from a general point of view, the flexible structure as a linear system represented by a finite-stage model of order N, and the external forces are undesired inputs of the system.

As it will be shown, the response to inputs is governed by the left eigenvectors of the system. In this paper, we propose a method to shape the closed-loop response by choice of the closed-loop eigenvalues and left eigenvectors of the system. If a left eigenvector of the system matrix is orthogonal to the columns of the input matrix $[E]$ corresponding to undesired inputs, then the response in this mode is zero. The purpose of this paper is to construct a closed-loop system whose dominant left eigenvectors possess this property. This method assumes that the undesired inputs are centralized on certain fixed points on the structure and the locations of these points are known.

From a mathematical point of view, the present problem is a special eigenspace assignment problem. For the early work in this field, pole placement was of primary interest.[2-4] Recently, the assignment of eigenvectors is treated as well as eigenvalues. A general condition of assignability of eigenvalues and eigenvectors using output feedback was given by Srinathkumar.[5] For the controllable and observable system described by equations

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\}$$

$$\{y(t)\} = [D]\{x(t)\}$$

if the matrices $[B]$ and $[D]$ are full rank, then $\max(m,r)$ closed-loop eigenvalues can be assigned, and $\max(m,r)$ eigenvectors (or reciprocal vectors by duality) can be partially assigned with $\min(m,r)$ entries in each vector arbitrarily chosen using output feedback. Juang et al.[6] showed how to generate an orthogonal basis that spans the admissible eigenvector space of the closed-loop system using the singular-value decomposition. An optimal set of eigenvectors corresponding to assigned eigenvalues were then determined in this subspace in order to obtain both minimum conditioning and control gains for the eigenvalue placement problem. Fennell[7] also used the flexibility in eigenvector assignment to reduce control system sensitivity to changes in system parameters with application to symmetric flutter suppression.

There are other authors who chose the desired eigenvectors based on the requirement of a given problem rather than optimal criteria mentioned previously. Andry et al.[8] used (right) eigenvector assignment to alter the transient response of the system for decoupling flight control. They showed first the subspace that represents the admissible eigenvectors of the closed-loop system, and then found, in this subspace, the closest eigenvectors to the desired ones. Other related work concerning eigenstructure assignment can be found in Fahmy and O'Reilly[9] who gave a general mathematical treatment of the eigenspace assignment problem, Zhang et al.,[10] who developed a modal method and applied it to vibration control of flexible structures, and Becus and Sonmez[11] who combined eigenstructure assignment with linear quadratic regulator (LQR) design of a multivariable control system. In the present paper, the eigenspace to be assigned consists of eigenvalues and left-hand eigenvectors; hence, the formulation and problem solution presented in this paper is quite different from those of the cited references.

In addition to eigenspace placement, the overall closed-loop stability of the control system must be considered in controller design. Using output feedback, only a reduced number of modes can be controlled, and for a high-order structure system the uncontrolled modes may destabilize the control system. To avoid this "control spillover," the concept of independent modal-space control will be very useful, based on which any number of modes can be controlled independently.[12,13] This method requires as many actuators as the number of controlled modes. In this paper, the analysis shows that, with the appropriate choice of the output matrix, controlled modes and uncontrolled modes can be totally decoupled. In addition, the number of controlled modes, which is equal to the number of the outputs, is not related to the number of actuators for the present method.

Problem Background

A linear system can be described by the equations

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [E]\{f(t)\} + [B]\{u(t)\} \tag{1}$$

$$\{y(t)\} = [D]\{x(t)\} \tag{2}$$

where $\{x\}\in R^N$, $\{f\}\in R^n$, $\{y\}\in R^r$, and $\{u\}\in R^m$ are state, input, output, and control vectors, respectively, and $[A]$, $[E]$, $[B]$, and $[D]$ are constant matrices of appropriate dimensions.

For a structural system, a natural way to define the system matrix $[A]$ is

$$[A]\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

where $[M]$, $[K]$, and $[C]$ are, respectively, mass, stiffness, and damping matrices. In this case, the state $\{x\}^T = \{p^T, v^T\}^T$ where $\{p\}$ is the displacement vector of the $N/2$ coordinates and $\{v\}$ is the vector of the corresponding velocities.

Assume that $[A]$ can be decomposed as

$$[A] = [\Phi][\Lambda][\Psi]^T \tag{3}$$

where $[\Lambda]\in C^{N,N}$ is a diagonal matrix of eigenvalues, $[\Phi]\in C^{N,N}$ is the matrix whose columns consist of the right eigenvectors of $[A]$, and $[\Psi]\in C^{N,N}$ is the matrix of left eigenvectors or "reciprocal vectors" of $[A]$; $[\Phi]$ and $[\Psi]$ are normalized as

$$[\Psi]^T[\Phi] = [I] \tag{4}$$

The columns of $[\Phi]$ are determined by solution of the standard eigenvalue problem as

$$[A - \lambda_i I]\{\phi_i\} = 0 \tag{5}$$

while the $\{\psi_i\}$ ($[\Psi] = [\cdots \psi_i \cdots]$) are determined from the adjoint problem

$$[A^T - \lambda_i I]\{\psi_i\} = 0 \tag{6}$$

These eigenvectors are closely related to the mode shapes defined in a standard modal analysis of the vibration system. Also not that whereas Eq. (4) implies that $[\Psi]^T = [\Phi]^{-1}$, these matrices are of high order and in general we need only a subset of $[\Phi]$ and $[\Psi]$ for the analysis to follow.

Assume initially that the control input $\{u(t)\} = 0$. Then the response of the system described by Eq. (1) due to input $\{f\}$ is

$$\{x(t)\} = e^{[A]t}\{x_0\} + \int_0^t e^{[A](t-\tau)}[E]\,\{f(\tau)\}\mathrm{d}\tau \tag{7}$$

or using the eigenvector matrices

$$\{x(t)\} = [\Phi]e^{[\Lambda]t}[\Psi]^T\{x_0\} + [\Phi]\int_0^t e^{[\Lambda](t-\tau)}\,[\Psi]^T[E]\{f(\tau)\}\mathrm{d}\tau \tag{8}$$

where $\{x_0\}$ is the initial condition of $\{x\}$ at $t = 0$.

Note that from Eq. (8), we can lead to the conclusion that the response to extraneous inputs $\{f\}$ can be eliminated if the columns of $[\Psi]$ are orthogonal to the columns of $[E]$. In the sequel we show how the feedback gain matrix can be chosen to achieve this.

Problem Statement

Our intent in this analysis is to choose the feedback gain matrix $[F]$, so as to cause the dominant left eigenvectors to be orthogonal to the columns of $[E]$. For the system of Eq. (1), we desire to find a feedback law that can reduce the effect of disturbances while stabilizing the system dynamics. For control assume an output feedback law

$$\{u\} = [F]\{y\} \tag{9}$$

The new system (closed-loop system) matrix $[\hat{A}]$ is defined as

$$[\hat{A}] = [A + BFD]$$

For the closed-loop matrix $[\hat{A}]$, denote the transformation matrices analog to Eq. (3) as

$$[\Lambda^{(d)}] = \mathrm{diag}\{\lambda_i^{(d)}\}, \quad [\Phi^{(d)}] = [\cdots \phi_i^{(d)} \cdots]$$
$$[\Psi^{(d)}] = [\cdots \psi_i^{(d)} \cdots]$$

where the columns of $[\Psi^{(d)}]$, $\{\psi_i^{(d)}\}$ are determined from Eq. (10).

$$[A^T + [BFD]^T - \lambda_i^{(d)}I]\{\psi_i^{(d)}\} = 0, \quad \{\psi_i^{(d)}\}^T\{\phi_j^{(d)}\} = \delta_{ij} \tag{10}$$

The goal of this paper is to determine the gain matrix $[F]$ such that the dominant closed-loop left eigenvectors $\{\psi_i^{(d)}\}$ are approximately orthogonal to the known disturbance input matrix $[E]$.

Feedback Gain Matrix Computation

For a large structural system it is neither possible nor desirable to control all of the modes of a system. Assume that there are a total of r modes that we desire to control. Now since the modal vectors span the N-dimensional space, it is reasonable to write the desired (closed-loop) left eigenvectors as a linear combination of the original (open-loop) left eigenvectors. We will, in fact, restrict the closed-loop eigenvectors further by assuming the desired eigenvectors are a linear combination of a subset of "q" open-loop eigenvectors, which we arrange as the matrix $[\Psi_1] = [\psi_1, \ldots, \psi_q]$

$$\{\psi_i^{(d)}\} = [\Psi_1]\{p_i\} \qquad \text{for} \qquad i = 1,\ldots,r \tag{11}$$

This assumption allows us to use only the known or identified eigenvectors in the analysis and is justified by noting that the contribution of high-frequency (open-loop) modes to low-frequency (closed-loop) modes is generally negligible. This can be shown as follows. Consider the case where $q = N$ such that $[\Psi]_1 = [\Psi]$. Rewriting Eq. (10) as

$$[A^T - \lambda_i^{(d)}I]\{\psi_i^d\} = \{g_i\}, \text{ with } \{g_i\} = -[BFD]^T\{\psi_i^{(d)}\} \tag{12}$$

and substituting Eq. (11) into Eq. (12) and premultipling by $[\Phi]^T$ yields

$$[\Lambda - \lambda_i^{(d)}I]\{p_i\} = [\Phi]^T\{g_i\} = \{\bar{g}_i\} \tag{13}$$

The jth component of the vector $\{p_i\}$, p_{ji} is found from Eq. (13) as

$$p_{ji} = \frac{\bar{g}_{ji}}{\lambda_j - \lambda_i^{(d)}} \tag{14}$$

This implies that only when λ_j is close to $\lambda_i^{(d)}$ $\{\psi_j\}$ is important in the closed-loop model. The truncation error due to omission of p_{ji} with $j > q$ is allowable in practice, since it will be less than other possible errors such as model uncertainty and extraneous disturbances. According to our experience, the truncation error in general can be neglected, when $q > (1.5$ to $2.0)r$. In some particular case, if the desired eigenvalues and eigenvectors are carefully chosen, we can even take $q = r$. It must be noted that Eq. (11) does not mean the $[\Psi_1]$ spans the achievable left eigenvector space of the closed-loop system. The achievability of the desired left eigenvectors will be discussed later in this paper.

To determine the feedback gain matrix $[F]$ to achieve eigenvector placement, substitute Eq. (11) into Eq. (10) and premultiply by $[\Phi_1]^T$:

$$[\Lambda_1 + \Phi_1^T D^T F^T B^T \Psi_1 - \lambda_i^{(d)}I]\{p_1\} = 0 \tag{15}$$

where $[\Lambda_1]$ is an $(r \times r)$ diagonal matrix of open-loop eigenvalues of the controlled modes. Equation (15) can be written in a more convenient form:

$$[\lambda_i^{(d)}I - \Lambda_1]\{p_i\} = [\Phi_1^T D^T F^T B^T \Psi_1]\{p_i\} \qquad \text{for} \qquad i = 1,\ldots,r \tag{16}$$

Equation (16) is the fundamental equation that relates the old and new eigenvectors, eigenvalues, and the gain matrix $[F]$. Not that only the identified eigenvalues and eigenvectors are required in this formulation. We are interested in what assignments are possible plus, for a specified eigenspace, we need an algorithm to solve for the gain matrix $[F]$.

In condensed form, Eq. (16) can be rewritten as

$$[\hat{P}] = [Z][H] \tag{17}$$

where $[\hat{P}] = [\cdots \hat{p}_i \cdots]$, $\{\hat{p}_i\} = [\lambda_i^{(d)}I - \Lambda_l]\{p_i\}$, $[Z] = [D\Phi_1]^T[F]^T$, $[H] = [B]^T[\Psi^{(d)}]$. While the matrices $[Z]$ and $[H]$ are, respectively, $(q \times m)$ and $(m \times r)$, $[\hat{P}]$ is a matrix $(q \times r)$ with rank $[\hat{P}] \le \min(m,r)$ (since $q \ge r$).

It is well known that any (singular or regular) matrix can be decomposed as two arbitrary full-rank matrices,[14] and if one of them is given, another will be uniquely determined. For cases of practical interest, we consider $m \le r$. (In practice, the number of controls m is, in general, less than the number of outputs r.) With appropriate choice of the desired eigenvalues and eigenvectors, we can pick $[\hat{P}]$ such that rank $[\hat{P}] = m$, and both $[Z]$ and $[H]$ are full-rank matrices. $[H]$ is a known matrix, thus the matrix $[Z]$ can be uniquely determined from Eq. (17). Furthermore, $[D\Phi_1]$ is a known square matrix of order r and is assumed to be nonsingular. Hence, if $[Z]$ is known, the feedback gain matrix $[F]$ can be obtained as

$$[F] = [Z]^T[D\Phi_1]^\dagger \tag{18}$$

Alternative Form for the Matrix $[Z]$

If the matrix $[Z]$ were directly resolved from Eq. (17), the desired eigenvalues and left-hand eigenvectors should be chosen so that rank $[\hat{P}] = m$. Otherwise there may be none or an infinite number of solutions of $[Z]$. To avoid this problem, we need to formulate an algorithm that does not require this explicit calculation and leads to a unique solution of $[Z]$ under some criteria. Reconsider Eq. (11) and rearrange as

$$\begin{aligned}\{\psi_i^{(d)}\} &= [\Psi_1][\lambda_i^{(d)}I - \Lambda_1]^{-1}[\lambda_i^{(d)}I - \Lambda_1]\{p_i\} \\ &= [\Psi_1][\lambda_i^{(d)}I - \Lambda_1]^{-1}\{\hat{p}_i\} \\ &= [\hat{\Psi}_1]_i[Z]\{h_i\} \qquad \text{for} \qquad i = 1,\ldots,r \end{aligned} \tag{19}$$

where $[\hat{\Psi}]_i = [\Psi_1][\lambda_i^{(d)}I - \Lambda_1]^{-1}$, and $\{h_i\}$ is the ith column of $[H]$.

From Eq. (19) each element $\psi_{kr}^{(d)}$ of $\{\psi_r^{(d)}\}$ can be expressed as a linear combination of components z_{lj} of the unknown matrix $[Z]$. These $N \times r$ linear algebraic equations can be arranged in the matrix form

$$\{\psi^{(d)}\} = [T]\{z\} \tag{20}$$

where the vector $\{z\}$ contains the elements z_{lj} for $l = 1, \ldots, q$, $j = 1 \ldots, m$, of the unknown matrix $[Z]$; $\{\psi^{(d)}\}$ is a $N \times r$ vector containing all specified components of $(\psi_i^{(d)})$, $i = 1, \ldots, r$, and $[T]$ is a $(N \times r, m \times q)$ known rectangular matrix.

In general, this set will be overdetermined; however, a least-squares solution of Eq. (20) can be obtained as

$$\{z\} = [T]^\dagger\{\psi^{(d)}\} \tag{21}$$

where $[T]^\dagger$ is the pseudoinverse of $[T]$, generally obtained by a singular-value decomposition algorithm. From $\{z\}$ we may form $[Z]$. The feedback gain matrix $[F]$ is then derived from Eq. (18).

In Eqs. (18–21), it is supposed that r eigenvalues and $N \times r$ entries of left eigenvectors are assigned. This is in contradiction with the theorem shown by Srinathkumar.[2] In fact, Eq. (20) implies that if the vector $\{\psi^{(d)}\}$ formed from all entries of the desired left eigenvectors lies precisely in the subspace spanned by the columns of $[T]$, they will be achieved exactly. Otherwise, only the projection of $\{\psi^{(d)}\}$ onto the "achievability subspace" can be achieved. Denote $\{\psi_A^{(d)}\} = [T][T]^\dagger\{\psi^{(d)}\}$ as the achieved left eigenvector by using the feedback gain matrix calculated through Eqs. (18–20); it is the closest left eigenvector to the desired one in the least-square sense, i.e.,

$$\min J = \sum_{i=1}^{r} \|\{\psi_i^{(d)}\} - \{\psi_{iA}^{(d)}\}\|^2 \tag{22}$$

In many practical cases only certain components of $\{\psi_i^{(d)}\}$ are required to be specified. A typical situation is that each of the columns of $[E]$ contains only m dominant components. In this case where only m entries are to be specified for each left eigenvector, only those components specified are used in Eq. (20). The vector $\{\psi^{(d)}\}$ consists of the $r \times m$ specified entries, and $[T]$ becomes a square matrix of order $r \times m$. The feedback gain matrix derived from Eqs. (18–21) can guarantee these specified components to be totally achieved.

Stability of the Control System

Since only a reduced number of modes (r among N) are controlled, it is possible that spillover into uncontrolled modes might destabilize the control system. Porter and Crossley[15] define the mode-controllability matrix: $[Q] = [\Psi]^T[B]$. It is easily shown that the ith mode is controllable by the jth input, if and only if $q_{ij} \neq 0$. In this paper we extend the mode controllability for overall stability of the closed-loop system. It can be proved that $N - r$ uncontrolled higher modes will become uncontrollable by properly chosing the control influence matrix $[D]$. Partition the matrix of right eigenvectors of the open-loop system $[\Phi]$ into the two submatrices: $[\Phi] = [\Phi_C, \Phi_U]$, where $[\Phi_C]$ consists of the columns of $[\Phi]$ corresponding to the controlled modes, and $[\Phi_U]$ are eigenvectors of uncontrolled modes. If the matrix $[\Phi_U]$ is in the null-space of $[D]$, the $N - r$ eigenvalues of uncontrolled modes of the closed-loop system become uncontrollable, i.e., they are identical to those of the open-loop system. For a proof of this statement we note that from Eq. (10) the eigenvalues of the closed-loop system $\lambda_i^{(d)}$ are the roots of the characteristic equation

$$\det[\Lambda + \Phi^T D^T F^T B^T \Psi) - \lambda^{(d)} I] = 0 \tag{23}$$

Let

$$[F^T B^T \Psi] = [\hat{F}] = [\hat{F}_C, \hat{F}_U] \qquad \text{with} \qquad [\hat{F}_C] \in C^{r,r}$$

Considering the orthogonality of $[D]$ to $[\Phi_U]$

$$[\Phi^T D^T] = \begin{bmatrix} \Phi_C^T D^T \\ \Phi_U^T D^T \end{bmatrix} = \begin{bmatrix} \hat{D} \\ 0 \end{bmatrix}, \qquad [\hat{D}] \in C^{r,r} \tag{24}$$

Substituting Eq. (24) into Eq. (23)

$$\det\left\{\begin{bmatrix} \Lambda_C & 0 \\ 0 & \Lambda_U \end{bmatrix} + \begin{bmatrix} \hat{D} \\ 0 \end{bmatrix}[\hat{F}_C \hat{F}_U] - \lambda^{(d)} I\right\} = 0 \tag{25}$$

Equation (25) can be written in the equivalent form

$$\det[\Lambda_C + \hat{F}_C - \lambda^{(d)} I] \cdot \det[\Lambda_U - \lambda^{(d)} I] = 0 \tag{26}$$

Equation (28) shows clearly that $\lambda_i^{(d)} = \lambda_i$ for $i = r + 1, \ldots, N$. Since the open-loop system is presumed stable and the r eigenvalues of the controlled modes lay naturally in the left half of the complex plane, the stability of the control system is guaranteed.

The assumed form of the output matrix implies that the rows of $[D]$ are the linear combination of columns of $[\Psi_1]$, the left eigenvectors of the controlled modes of the open-loop system. It has to be mentioned that in this section we only give a mathematical criterion to decouple controlled modes with uncontrolled modes; however, how to obtain such an output matrix $[D]$ in practice must be investigated case by case.

Control Implementation on Flexible Structures

In this section, we return to flexible structures and several practical problems of control implementation are discussed.

Data Base

The data base required by the proposed method consists of the controlled eigenvalues and eigenvectors of the original structure (open-loop system). These modal parameters can be either from an analytical model of the structure, such as a finite-element model, or from modal identification of the desired modes.

The advantages of using the modal identification are that this method avoids the modeling errors due to discretization and uncertainty in structural characteristics, and also that an on-line modal identification implement can be effective in cases where the system exhibits time variation or nonlinear characteristics, which may produce significant variation in modal characteristics from nominal design. Considering the recent advances in the modal identification area,[16-21] the proposed method can be used to build up an integrated identification/control procedure in a realistic flexible structure problem.

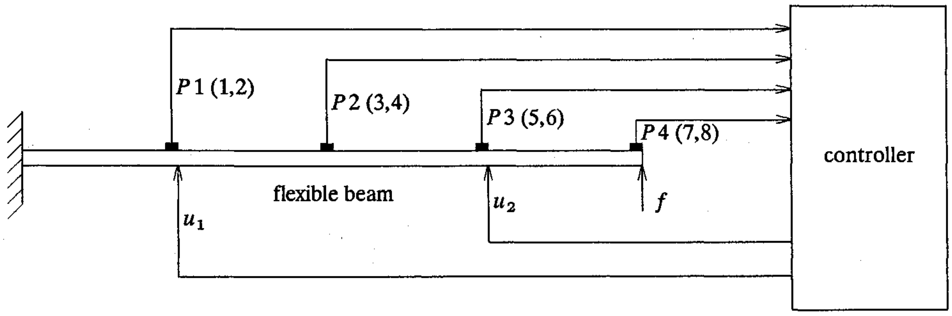

Fig. 1 Test structure.

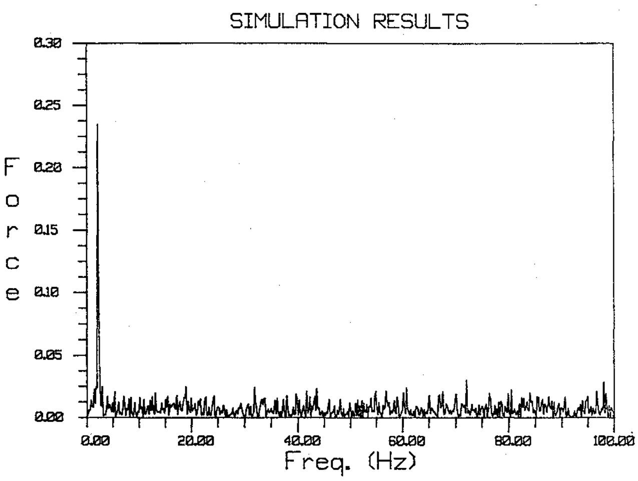

Fig. 2 Power spectrum of external force configuration 1.

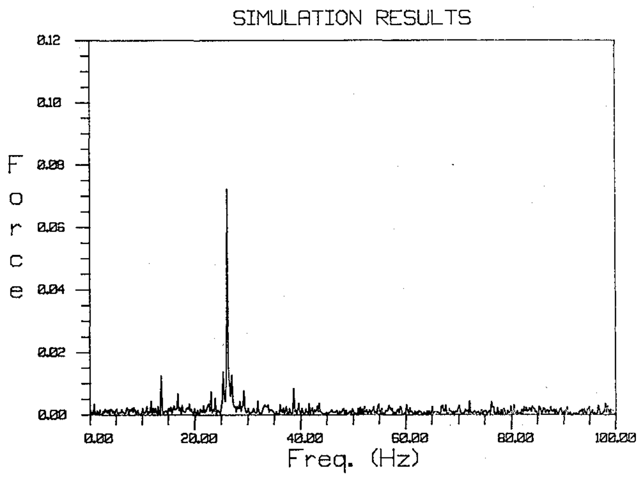

Fig. 3 Power spectrum of external force configuration 2.

Selection of the Controlled Modes

Since only r modes can be controlled among total N modes, one has to first decide which modes are to be controlled. Since the excitation of higher-order modes needs more energy than lower-order modes, a reasonable approach is to consider the lowest modes to be controlled. A singular perturbation technique[22] or a balanced realization technique[23] also can be used to select the controlled modes. For the present problem the selection of the controlled modes depends on the external forces. If the power spectrum of the external forces can be approximatively estimated, the controlled modes must cover the frequency bandwidth where the main energy of the external forces is concentrated. The numerical example given in the next section will show how it is important to select controlled modes differently for different external force configurations.

Numerical Example

As an illustration, let us consider a uniform beam fixed at one end and free at the other. A finite-element model of this beam is composed of four elements having a total of 8 degrees of freedom (translation and rotation at each node). The actuators are implemented at point 1 (P1) and at 3 (P3). A single external force is applied at the free end of the beam [at point 4 (P4) in the direction of translation, which is arranged as degree of freedom 7]. The force vector $\{f\}$ and the matrix $[E]$ have the following forms:

$$\{f\} = \begin{Bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ f \\ 0 \end{Bmatrix}, \qquad [E]\begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$

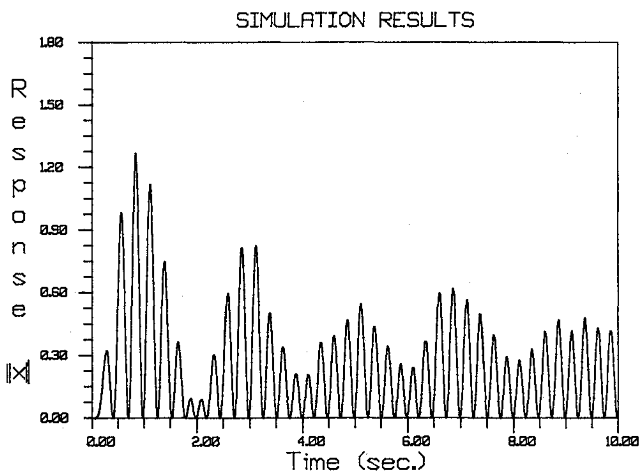

Fig. 4 Global response of the structure without control (external force configuration 1).

The desired left eigenvector of the controlled modes $\psi_i^{(d)}$ is chosen such as

$$\{\psi_i^{(d)}\}^T[E] = \{x \cdot \cdot x 0 x\}$$

where the 0 corresponds to the only nonzero element of the force vector $\{f\}$.

Two different configurations of the external force are considered:

1) The external force is the sum of sinusoidal function whose frequency is 2 Hz, and a random (white noise) function (see Fig. 2).

2) The frequency of the sinusoidal function is at 25 Hz, and the random function is unchanged from 1 (see Fig. 3).

In the first configuration, only the two lowest modes are controlled. Figure 4 shows the norm of the response to the external force described previously without applying the active control. The forced response vector $\{x(t)\}$ contains only the translations of the elements, and the norm is defined as

$$\|x\| = \sqrt{\{x(t)\}^T\{x(t)\}} \tag{27}$$

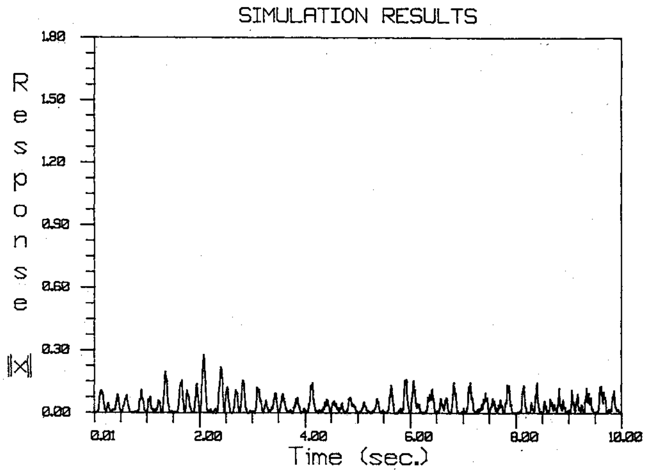

Fig. 5 Global response of the structure under control (two modes) (external force configuration 1).

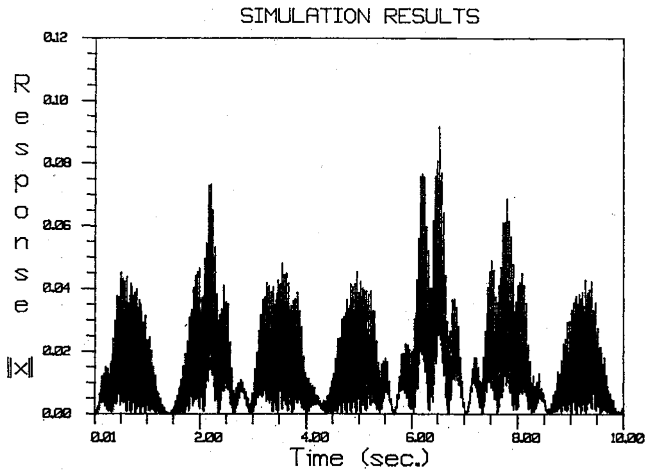

Fig. 6 Global response of the structure without control (external force configuration 2).

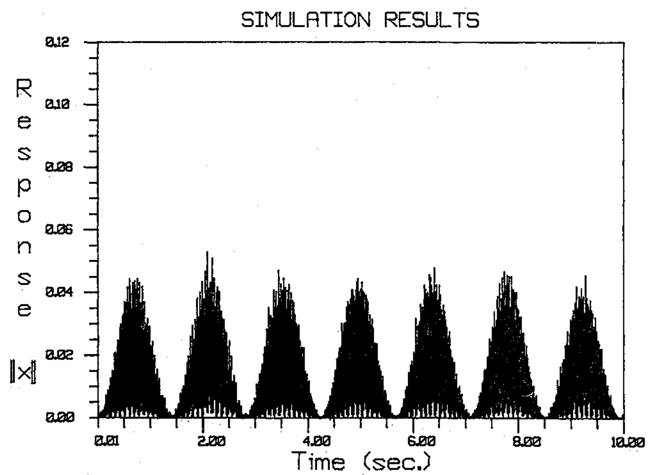

Fig. 7 Global response of the structure under control (external force configuration 2).

Figure 5 shows $\|x\|$ with the active control. Comparing Fig. 5 with Fig. 4, we see that the global vibration of the structure is significantly reduced by using the proposed method. Note that since the main energy of the external force is at 2 Hz, the contribution of higher modes on the response is very weak, and the effect of the external force can be greatly reduced by shaping the two lowest modes whose natural frequencies are 1.5 and 9.5 Hz, respectively.

In the second configuration, we control successively the two lowest modes and then the four lowest modes. The results without and with control effect are shown in Figs. 6–8. From these figures we see now that controlling two modes is not enough to reduce the vibration of the structure, because these modes do not cover the frequency bandwidth where the energy of the external force is concentrated.

In this numerical example, assume the full state vector is measured, and each output is a linear combination of the states. The output matrix $[D]$ is chosen as $[D] = [\Psi_1]^T$, and so the controlled modes are completely decoupled from uncontrolled modes. The original, desired, and achieved eigenvalues of both cases are represented in Tables 1 and 2.

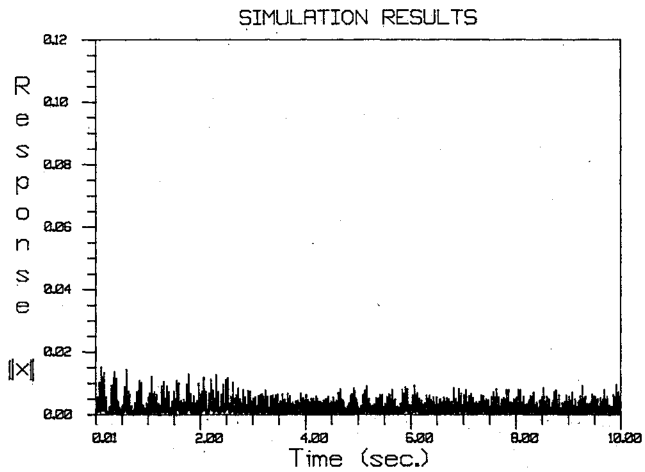

Fig. 8 Global response of the structure under control (four modes) (external force configuration 2).

Table 1 Original, desired, and achieved eigenvalues

	Eigenvalues[a]		
Mode no.	Original	Desired	Achieved
1	$-0.446+j9.484$	$-0.750+j30.00$	$-0.750+j30.00$
2	$-1.978+j59.48$	$-1.250+j60.00$	$-1.250+j60.00$
3	$-0.002+j167.8$	—	$-0.002+j167.8$
4	$-2.096+j331.0$	—	$-2.096+j331.0$
5	$-0.009+j615.7$	—	$-0.009+j615.7$
6	$-1.758+j988.8$	—	$-1.758+j988.8$
7	$-0.077+j1568.0$	—	$-0.077+j1568.0$
8	$-0.020+j2572.0$	—	$-0.202+j2572.0$

[a]Two modes assigned.

Table 2 Original, desired, and achieved eigenvalues

	Eigenvalues[a]		
Mode no.	Original	Desired	Achieved
1	$-0.446+j9.484$	$-0.750+j30.00$	$-0.750+j30.00$
2	$-1.978+j59.48$	$-1.250+j60.00$	$-1.250+j60.00$
3	$-0.002+j167.8$	$-0.750+j190.0$	$-0.750+j190.0$
4	$-2.096+j331.0$	$-1.250+j350.0$	$-1.250+j350.0$
5	$-0.009+j615.7$	—	$-0.009+j615.7$
6	$-1.758+j988.8$	—	$-1.758+j988.8$
7	$-0.077+j1568.0$	—	$-0.077+j1568.0$
8	$-0.020+j2572.0$	—	$-0.202+j2572.0$

[a]Four modes assigned.

Concluding Remarks

A special eigenspace assignment method is presented. The main application of the proposed method is to suppress certain undesired inputs of linear systems. The advantage of this method is that it does not need to measure the undesired inputs. Hence, when the undesired inputs are applied at specified known locations, this method can be used. The numerical example presented in this paper illustrates how the global vibration of a flexible structure can be significantly reduced.

Using this method, the number of controlled modes is equal to the number of outputs r, which depends on the dimension of the controlled modal space of the open-loop system. From a practical point of view, this means that with few actuators, quite a large number of modes can be controlled, which is another advantage of the proposed method.

It has to be noted that for application of the proposed method, the selection of the location of the actuators is important, especially for large complex structural systems. If the actuators are placed at node points of a mode, this mode cannot be changed by the control forces. A bad distribution of the actuators in the structure can also cause an ill-conditioning of the matrix $[T]$. In contrast, a suitable distribution of the actuators on the structure can improve the performance of the control system. Hence, before the implementation of the control system, a sensitivity analysis is advised to guide the selection of the location of the actuators.

For the present method, the stability is guaranteed by choosing an appropriate output matrix whose rows should be a linear combination of the left eigenvectors of the controlled modes of the open-loop system. This implies that the full state vector has to be measured or reconstructed which might be a critical point for the application on large structural systems. However, for many systems, the required excitation energy and the inherent damping of the higher modes mean in practice that these modes may often be neglected. Further, by restricting the bandwidth of the controller and including all modes below this cutoff frequency, the control system will maintain stability. The direct system parameter identification technique provides a low-order model governing the response of the system to inputs in a given frequency bandwidth.[24] Further work to provide a more general stability condition would be desired.

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